Integration By Parts
Case Study: The Antiderivative Of A Natural Log

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We want to solve the following indefinite integral (i.e. antiderivative)...

$$I = \int \ln(\theta)\,\delta\theta \tag{1}$$

We will define the function $F(\theta)$ to be the following equation...

$$F(\theta) = \ln(\theta) \text{ ...such that... } F'(\theta) = \frac{\delta \ln(\theta)}{\delta\theta} = \theta^{-1} \tag{2}$$

We will define the function $G(\theta)$ to be the following equation...

$$G(\theta) = \theta \text{ ...such that... } G'(\theta) = \frac{\delta\theta}{\delta\theta} = 1 \tag{3}$$

Using Equations (2) and (3) above, we can rewrite Equation (1) above as...

$$I = \int F(\theta)\,G'(\theta)\,\delta\theta \tag{4}$$

Note that using integration by parts we can rewrite Equation (4) above as... [1]

$$I = F(\theta)\,G(\theta) - \int F'(\theta)\,G(\theta)\,\delta\theta \tag{5}$$

Using Equations (2) and (3) above, the solution to Equation (5) above is...

$$\begin{aligned} I &= \theta\,\ln(\theta) - \int \theta^{-1}\theta\,\delta\theta \\ &= \theta\,\ln(\theta) - \int \delta\theta \\ &= \theta\left(\ln(\theta) - 1\right) \end{aligned} \tag{6}$$

References

[1] Gary Schurman, *Integration By Parts - The Mathematics And A Simple Example,* January, 2020